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Comparative Analysis of Different Control Schemes in Delta Domain Using Time Moments

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ABSTRACT

Traditionally, discrete-data sampled data systems are represented using shift-operator parameterization. Such parameterization was not suitable at high sampling rates. An alternative parameterization using the so-called delta operator maintains the close correspondence to its continues-time counterpart at fast sampling rates. This paper deals with the application of time moment estimation and adaptive control schemes. In the fast sampling limit, the delta operator model tends to the analog dynamic system model. This intrinsic property of the delta operator model unifies continues and discrete time control engineering. Comparative analysis results are showed the usefulness of the scheme.

Keywords: *Model Reference Adaptive Control Delta Domain Sampling Rates*.

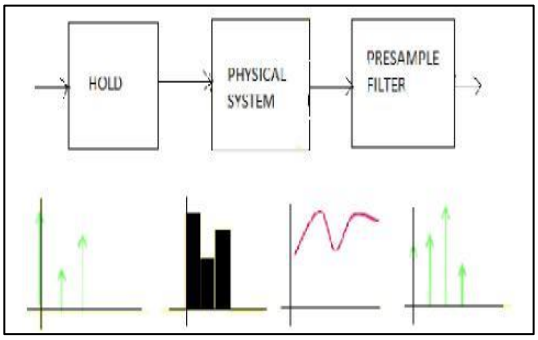
1.0 Introduction

An adaptive controller is an “intelligent controller” which can modify its behavior to changes to dynamics of the process and characteristics of the disturbances. According to the Webster’s dictionary [1],” to adapt” means to change to conform to new circumstances. So, adaptive controller is a mechanism for adjusting the parameters. Simply, adaptive control system consist two closed loops. One loop is a normal feedback loop in which controller and the plant comes and other loop is a parameter adjustment loop. Control of fully known deterministic, linear time invariant dynamic systems have received wide attention for many decades and a lot of study and surveys have been performed. Among the different types of adaptive schemes traditionally [2] four such schemes namely self-oscillating, gain scheduling, auto tuning, model reference adaptive control (MRAC) are in wide use. Here we have used Model Reference Adaptive control (MRAC) [3] framework to attain performance characteristics. In system identification the moment matching technique is a well proven technique. But it has been used in offline system identification. But it has been used in [4-7]. Using these schemes simulation results are presented. The simulation results are obtained by moment matching control schemes available from [7] namely (a) Plant time moment controller scheme (PTCS) (b) Plant time moment controller with feedback scheme (PTCFS) & the conclusions are drawn.

Section II gives a brief introduction to Sampled Data Models for Linear Deterministic Systems in delta domain. In Section III we give a brief introduction about delta-operator. In Section III we detail the estimation of time moments of different control schemes. In Section IV we apply delta-operator in different control schemes like PTCS and PTCFS. In Section VII conclusion are drawn with the future scope.

2.0 Sampled Data Systems in Delta Domain

Consider a sampled data system with input, where is the sampling period and k is an indexing discrete-time parameter, which is processed by a digital to analog (D/A) converter to give the continues-time input as shown in below figure.



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Usually, the D/A is designed in such a way, that the value of u_c (t) is held constant between samples, known as zero-order hold (ZOH). The continuous-time output $y_c\ (t)$ is then sampled with a period Δ using an analog to digital (A/D) converter to give the sampled output $y(K\Delta)$. In practice one must Prefilter the continuous-time output to avoid aliasing problems.

2.0 Definition

The δ - operator is defined in the time-domain as Which demonstrates the close relationship between the discrete-time δ -operator and the continuous-time

$$\delta = \frac{q-1}{\Delta} \quad \ldots\ldots\ldots(1)$$

Where Δ is the sampling period and q is the forward shift operator. Operating δ on a differential signal $x(t)$ gives

$$\delta x(t) = \frac{x(t+\Delta)-x(t)}{\Delta} \quad \ldots\ldots (2)$$

It is straight forward to see that

$$\lim \Delta \to 0 \ \delta x(t) = \frac{d}{dt}x(t) \quad \ldots. (3)$$

Which demonstrates the close relationship between the discrete-time δ-operator and the continuous-time differential operator $\frac{d}{dt}$ at high sampling rates. Note that (1) is a simple linear transformation and thus system modeling using δ -operator parameterization offers exactly the same flexibility as q-operator parameterization, i.e. the class of describable systems is not changed.

Similar relation exists in the complex domain as well. The delta transform operator *y* is defined as

$$\gamma = \frac{z-1}{\Delta} \quad \ldots\ldots(4)$$

Where z is the complex domain translorm operator tor the discrete-time system, like the Laplace transform operator of continuous-time system.

3.0 Time Moments Estimation

Consider a plant in delta domain as $G(\gamma)$. Let its impulse response be $g(k\Delta)$. Let y be the output of G excited by u. Then

$$Y(\gamma) = G(\gamma)U(\gamma) \quad \ldots\ldots (5)$$

Since G is asymptotically stable, it permits a series expansion in terms of its time moments $\{k_i\}$ as

$$G(\gamma) = k_0 + k_1\gamma + k_2\gamma^2 + \ldots = \sum_{k=0}^{\infty} k_i\gamma^i \quad \ldots. (6)$$

The problem is to obtain estimates of $\{k_i\}$ from on-line measurements, up to the current time of u and y. Consequently, U(γ) is defined as follows:

$$U(\gamma) = \Delta\sum_{k=0}^{\infty} U(k\Delta)(1+\Delta\gamma)^{-k}$$
$$= \Delta[U(0) + U(\Delta) + U(2\Delta) + \ldots]$$
$$- \Delta^2\gamma[U(\Delta) + 2U(2\Delta) + \ldots]$$
$$+ \Delta^3\gamma^2[U(\Delta) + 3U(2\Delta) + \ldots] + \ldots\ldots\ldots (7)$$
$$= \Delta\sum_{k=0}^{\infty} U(k\Delta) - \Delta\gamma\sum_{k=0}^{\infty}(k\Delta)U(k\Delta) +$$
$$\Delta\gamma^2\sum_{k=0}^{\infty} k(k+1)\Delta^2 U(k\Delta) + \ldots\ldots\ldots (8)$$

Similarly,

$$Y(\gamma) = \Delta\sum_{k=0}^{\infty} Y(k\Delta)(1+\Delta\gamma)^{-k}$$
$$= \Delta[Y(0) + Y(\Delta) + Y(2\Delta) + \ldots]$$
$$- \Delta^2\gamma[Y(\Delta) + 2Y(2\Delta) + \ldots]$$
$$+ \Delta^3\gamma^2[Y(\Delta) + 3Y(2\Delta) + \ldots] + \ldots\ldots\ldots (9)$$
$$= \Delta\sum_{k=0}^{\infty} Y(k\Delta) - \Delta\gamma\sum_{k=0}^{\infty}(k\Delta)Y(k\Delta) +$$
$$\Delta\gamma^2\sum_{k=0}^{\infty} k(k+1)\Delta^2 Y(k\Delta) + \ldots\ldots (10)$$

Equating Y(γ)=G(γ)U(γ)

We get,

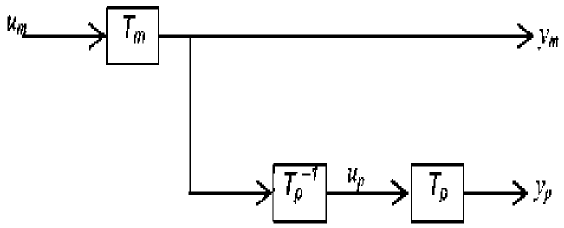
$$k_0 = \frac{\sum_{k=0}^{\infty} Y(k\Delta)}{\sum_{k=0}^{\infty} U(k\Delta)} \quad \ldots\ldots\ldots (11)$$

$$k_1 = \frac{-\Delta\left[\sum_{k=0}^{\infty} kY(k\Delta) - k_0\sum_{k=0}^{\infty} kU(k\Delta)\right]}{\sum_{k=0}^{\infty} U(k\Delta)} \quad \ldots\ldots\ldots (12)$$

$$k_2 = \frac{\Delta^2\sum_{k=0}^{\infty} k(k+1)Y(k\Delta) - k_0\Delta^2\sum_{k=0}^{\infty} k(k+1)U(k\Delta) + k_1\Delta\sum_{k=0}^{\infty} kU(k\Delta)}{\sum_{k=0}^{\infty} U(k\Delta)} \quad \ldots\ldots\ldots (13)$$

4.0 Control Design in Delta Domain

a. Plant Command Modifier Scheme in Delta Domain



The plant command modifier scheme as proposed in [6] has been modified in this section in the delta domain with the goal to study the control scheme and the application of the online estimation scheme.

The basis on which this PCMS is built is shown in the figure1. Suppose *Tp*–1 is available. Then *up* could have been obtained as to get $y_p = y_m$. Obviously, *Tp*–1 will be unstable in the event of T_p having non-minimum phase zero(s). To overcome this problem, a method involving time moments has been proposed.

$$u_p = (T_p^{-1}\ T_m)u_m \ldots\ldots\ldots\ldots\ldots(14)$$

Let $\{k_{i,p}\}$ denote the time moments of the unknown plant *Tp*. Then

$$T_P(\gamma) = \sum_{i=0}^{\infty} k_{i,p}\gamma^{i} \ldots\ldots\ldots\ldots(15)$$

Regardless of the number of its zero in excess over its poles, T_p^{-1} permits an expansion in terms of its time moments $\{q_i\}$ as

$$T_p^{-1}(\gamma) = \sum_{i=0}^{\infty} q_{i,p}\gamma^{i} \ldots\ldots\ldots \quad (16)$$

Now, $T_p T_p^{-1} = 1$ leads to

$$F = \begin{bmatrix} f_0 & 0 & 0 \ldots\ldots\ldots & 0 \\ f_1 & f_0 & 0 \ldots\ldots\ldots & 0 \\ & & & \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

$$\sum_{i=0}^{\infty} k_{i,p}\gamma^{i} \sum_{i=0}^{\infty} q_{i,p}\gamma^{i} = 1 \qquad \ldots\ldots(17)$$

Collecting the coefficients of like powers of γ yields

$k_{0,p}q_0 + (k_{1,p}q_0 + k_{0,p}q_1)\gamma + (k_{2,p}q_0 + k_{1,p}q_1 + k_{0,p}q_2)\gamma^2 +$ ------ = 1,

and hence

$k_{0,p}\, q_0 = 1,$

$k_{1,p}\, q_0 + k_{0,p}\, q_1 = 0,$

$k_{2,p}\, q_0 + k_{1,p}\, q_1 + k_{0,p}\, q_2 = 0,$

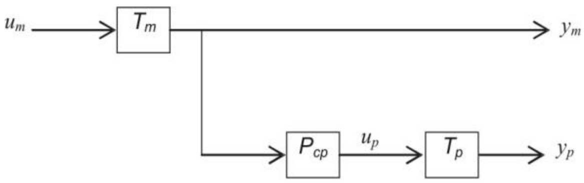
and so on. This is same as

$k_{0,p}\, q_0 = 1,$

$$\sum_{j=0}^{i} k_{i-j,p}\, q_j = 0; \quad i = 1, 2, \text{------} \quad \ldots\ldots (18)$$

b. Padé-adapted Plant Command Modifier Scheme in Delta Domain

Figure 2: Schematic for PPCMS



Here $P_c(\gamma) = f_0 + f_1\gamma + \ldots\ldots\ldots + f_\eta\gamma^\eta$

With

$$f_0 = \frac{1}{\hat{k}_{0,p}}, \quad f_i = -\frac{\sum_{j=0}^{i-1}\hat{k}_{i-j,p}f_j}{\hat{k}_{0,p}};$$

$i = 1,2,\ldots\,\eta$ ………………………………..(19)

We now proceed to analyze the implications of this representation with $\mu \geq \nu$

$P_c(s) = f_0 + f_1\gamma + \ldots\ldots\ldots + f_\eta\gamma^\eta$

=

through $b_0 + b_1\gamma + \ldots + b_\nu\gamma^\nu$

Without loss of generality, A(γ) can be chosen monic, i.e. $a_\mu = 1$ set $\mu = \nu = \eta$. Cross multiplying and equating the coefficients of like powers of γ leads to

$Fa = b,$ …………… (20)

Where

$a = [a_0\ a_1 \ldots\ldots a_{\eta-1}]^T$ and

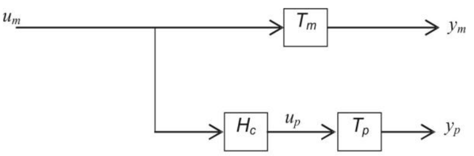
$$b = [b_0\ b_1 \ldots\ldots b_{\eta-1}]^T$$

The identities obtained by equating the coefficients of γ^i, i= η+1, η+2, …,2η have been neglected. The consequence is that with the compensator taking the form

$$P_{cp} = B/A \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(21)$$

c. Plant Time Moment Controller Scheme in Delta Domain

Figure 2: Schematic for PTCS



Here

$$H_c(\gamma) = \sum_{i=0}^{\infty} h_i \gamma^i \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(22)$$

$$\text{and } T_m(\gamma) = \sum_{i=0}^{\infty} k_{i,m} \gamma^i \quad \ldots\ldots\ldots\ldots\ldots\ldots(23)$$

Along the same lines as in PCMS, set

$$\sum_{i=0}^{\infty} k_{i,m} \gamma^i = \sum_{j=0}^{\infty} k_{j,p} \gamma^j \sum_{l=0}^{\infty} h_l \gamma^l \ \ldots (24)$$

Truncating this identity up to i = η on the l.h.s., replacing $\{k_{j,p}\}$ by their estimates $\{\hat{k}_{j,p}\}$ and following the steps similar to those in PCMS finally results in

$$\sum_{j=0}^{i} k_{i-j,p}\ h_j = k_{i,m}; \quad i = 1,2,\text{------}\ \eta\ldots\ldots\ldots (25)$$

$$\hat{K}_p h = k_m \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (26)$$

Where

$$\hat{K}_p = \begin{bmatrix} \hat{k}_{0,p} & 0 & 0 \text{ ------- } 0 \\ \hat{k}_{1,p} & \hat{k}_{0,p} & 0 \text{ ------- } 0 \\ | & | & \\ \hat{k}_{\eta,p} & \hat{k}_{\eta-1,p} & \hat{k}_{0,p} \end{bmatrix}$$

$$h = [\,h_0 h_1 \text{ ---------- } h_{\eta-1} 1]^T \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (27)$$

$$\text{and } k_m = [k_{0,m} k_{1,m} \text{ ------ } k_{\eta-1,m} k_{\eta,m}]^T \ldots\ldots\ldots\ldots (28)$$

As seen earlier and, hence, $\hat{k}_{0,p}$ are finite and nonzero. The matrix K_p is non singular. We can solve for a unique *h*, which is necessary and sufficient to match the first (*η*+1) time moments. The next step is to realize H_c by following the design procedure used in Section IV (b). pk,0ˆ

$$H_c(\gamma) = h_0 + h_1\gamma + \text{-} + \gamma^{\eta} = \frac{B(s)}{A(s)} = \ldots(29)$$

$$= \frac{b_0 + b_1\gamma + \text{---} + b_v\gamma^{v}}{a + a\ \gamma + \text{-------} + a\ \gamma^{\mu}}$$

through $\gamma^{\mu+v}$

Set a_μ=1. Cross multiplying and equating the coefficients of like powers of γ yields.

$$Ha = b\ldots\ldots. (30)$$

Where

$$H = \begin{bmatrix} h_0 & 0 & 0 \text{ -------- } 0 \\ h_1 & h_0 & 0 \text{ -------- } 0 \\ \text{---------------------} & & \\ h_\eta & h_{\eta-1} & \text{-------- } h_0 \end{bmatrix}$$

$$a = [\,a_0\ a_1 \text{ ---------- } a_{\eta-1}\ 1]^T \quad \text{and} \quad b = [b_0\ b_1 \text{ ------ } b_{\eta-1}\ b_\eta]^T\ldots$$

Specify A as done in Section V (b). Then compute b.

d. Plant Time Moment Controller with Feedback Scheme in Delta Domain

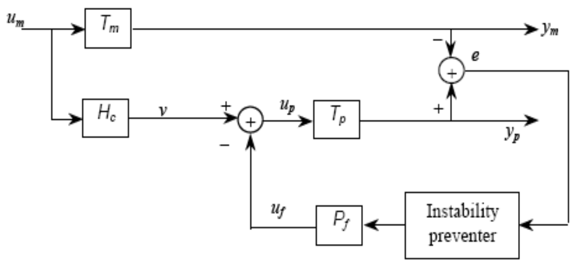


Figure 4. Implementation of PTCFS

In this PTCF scheme, *v* of Fig.4 is generated as

$v = Hcum$……………………………… (31)

5.0 Results

Fig 5: Minimum Phase SISO Plant with no Control

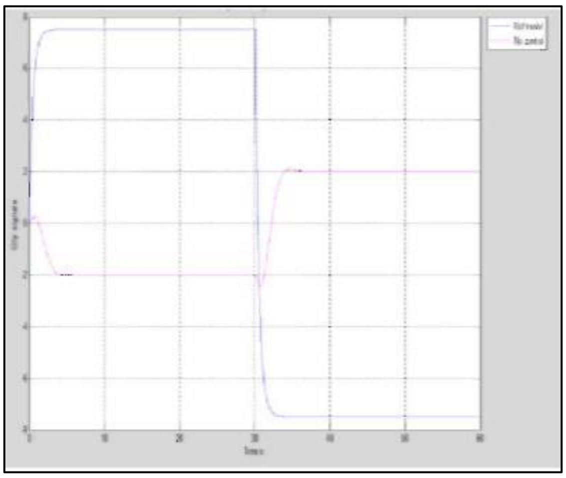


Fig 6: Minimum Phase SISO Plant with Different Controls; Matching One Moment

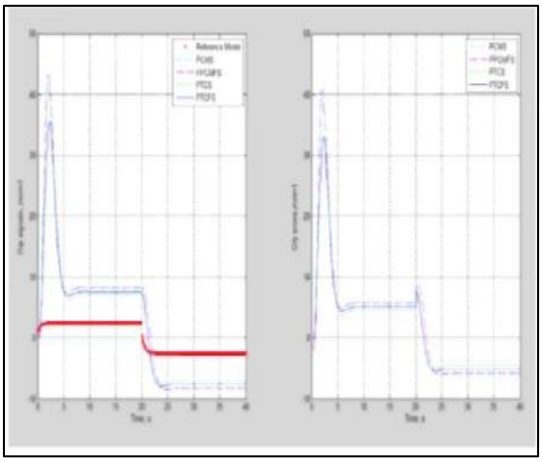


Fig 7: Minimum Phase SISO Plant with Different Controls; Matching Two Moments

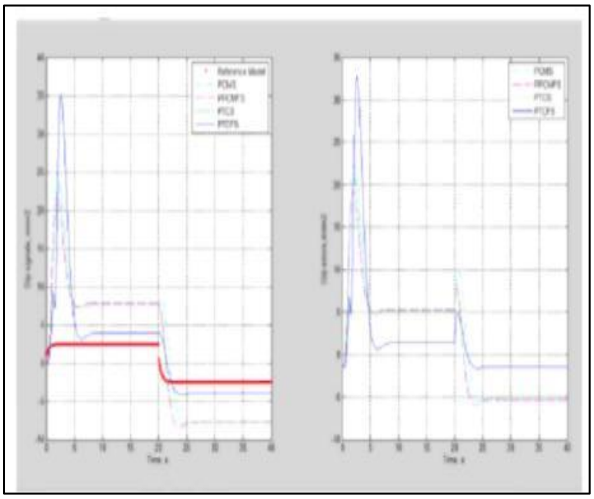


Fig 8: Minimum Phase SISO Plant with Different Controls; Matching Three Moments

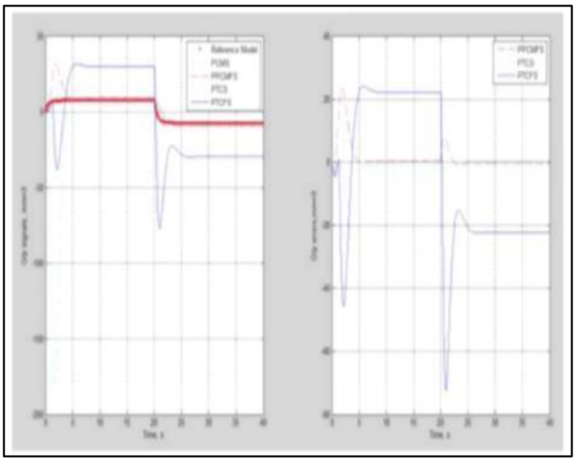
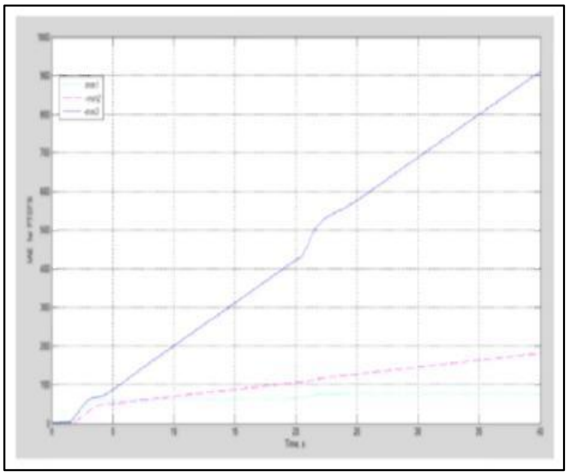


Fig 9: Minimum Phase SISO Plant with PTCFS; IAE Comparison with One Two and Three Moments Matched



From the above all control schemes figures, we analyze that The initial overshoot in the first half cycle is reduced in the two moment matching case with respect to one moment matching but the model following time is not delayed in the positive half cycle of the input. This may be due to the reason of adding an additional time moment. Although, the performance in the negative half cycle of the input remains almost the same for all the different moments matched. When the three moments are matched the initial overshoot in the positive half cycle for the different controls get further reduced. But again the model following time is the same and it is not delayed. To compare the performances of the control schemes the performance measuring index used as integral absolute error (IAE). A head to head comparison of the two schemes is made, the number of moments matched considered is equal. From the above comparison we again get the information that the PTCF scheme again performs the best because it has the least IAE.

6.0 Conclusions

The various control schemes proposed, PTCFS performs to be the best because it has least IAE which is evident from the results obtained from the figure 9. In the PTCFS scheme, there is a steady improvement with every additional time moment matched, which is evident from the figures. As a future scope of work we can extend this discussion for multi-variable systems as well. The model matching controller design procedures developed in this work are not directly applicable to nominally unstable systems. This aspect merits further investigation.

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